A scalar field modelling of the rotational curves of spiral galaxies

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Abstract

In a previous work [1], we have modelled the rotation curves (RC) of spiral galaxies by including in the equation of motion dynamical terms from an external real self-interacting scalar field, ϕ, minimally coupled to gravity and which respects the equivalence principle in the absence of electromagnetic fields. This model appears to have three free parameters : the turnover radius, r_0, the maximum rotational velocity, $v_{max} = v(r_0)$, plus a strictly positive integer, n. Here, the coupling of the ϕ-field to other kinds of matter is emphasized at the expense of its self-interaction. This reformulation presents the very advantageous possibility that the same potential may be used now for all galaxies. New correlations are established.

1 Introduction

The equation of motion of a neutral test body reads in the presence of the ϕ-field

$$\frac{du^\mu}{ds} + \Gamma^\mu_{\alpha\beta}\, u^\alpha\, u^\beta = \frac{d\phi}{ds}\, u^\mu - \partial^\mu \phi, \tag{1}$$

where the geodesic equation is recovered in the case of a non variable ϕ-field. In the weak fields and low velocity limit, assuming spherical symmetry and circular orbits, the rotational velocity, v, at radius r is then given by

$$\ln{(rv)} = \ln J + \phi, \tag{2}$$

where J is a constant which would represent the angular momentum per unit mass if the ϕ-field were not present within the galaxy[1]. Assuming that the excitation of the ϕ-field is very small compared to its vacuum expectation value, ϕ_{vev}, the equation of the ϕ-field reads in the first approximation

$$\partial_\mu \, \partial^\mu \phi = - \, g \, \chi \, \phi_{vev} \, T, \tag{3}$$

where T denotes the trace of the energy-momentum tensor of the source of the ϕ-field, $\chi = 8\pi G/c^4$ is the Einstein gravitational constant and g a universal dimensionless coupling constant. Hence, one gets for a static spherical matter distribution

$$\frac{d^2\phi}{dr^2} \; + \; \frac{2}{r}\,\frac{d\phi}{dr} = g \, \frac{8\pi G}{c^2} \, \phi_{vev} \, \rho, \tag{4}$$

where $\rho = \rho_{bulge} \, + \rho_{disk} \, + \, \rho_{halo}$ denotes the mass density of the matter fields other than the ϕ-field itself. Assuming in addition a sufficiently thin disk (stellar plus gaseous disks) and a (quasi-isothermal) spherical dark halo with mass density such that

$$\rho_{halo} \propto 1/r^{2\,+\,1/n}, \tag{5}$$

the static spherical solution of ϕ is found proportional to $r^{-1/n}$ (up to the vacuum expectation value) within the galaxy out of the bulge. Hence, the rotational velocity reads

$$v = v_{max} \, G_n(r/r_0), \tag{6}$$

where the functions G_n are defined as follows

$$G_n(x) = \frac{1}{x} \, \exp\left[\, n(1 - x^{-1/n}) \,\right]. \tag{7}$$

For our purpose, $n \geq 2$. Indeed, this is one of the two necessary conditions for the ϕ-field mimics a dark matter mass profile ($\propto r^{1-1/n}$).

[1]Here, we just deal with the tangential equation since we have assumed circular orbits. The radial equation would yield the radial velocity but, it is found negligible with respect to the tangential velocity, out of the bulge.

2 Results

As can be seen in figure 1, the greater is the integer n, the steepest is the curve $y = v/v_{max}$ versus $x = r/r_0$ for $x < 1$ and the flatest it is for $x > 1$. Hence, the steepest is a RC below the turnover radius, the flatest it should be beyond. Figure 2 shows some fits to individual rotation curves (from the samples of Rubin *et al.* [2, 3], van Albada *et al.* [4], Lake and van Gorkom [5]). This is achieved by using the least-squares fit to search the parameters a and $b = \ln J$ that yield the maximum square, R^2, of the correlation coefficient for the relation $\ln(rv) = ar^{-1/n} + b$. As yet, based on the study of a hundred spirals, it is found that a is always negative (this is the other necessary condition for the ϕ-field mimics a dark matter mass profile) whereas b is always positive. In addition, one gets approximately the following statistics : $n = 5$ for 20% of the spirals, $3 \leq n \leq 6$ for almost a half of them and $3 \leq n \leq 12$ for 80% of them. Figure 3 shows our new result : There is a strong correlation between the coefficients a and b. Though displayed here for only fifteen spirals, this remains true for the whole sample of a hundred spirals we have studied hitherto. Hence, if a natural theoretical explanation is found in the future, only two independent parameters will be needed to fit the RC with the proposed model one of which is just an integer.

3 Conclusion

It is possible that long range scalar fields external to gravity play a significant role not only at the cosmological level but also at the scale of galaxies or even the solar system [6].

References

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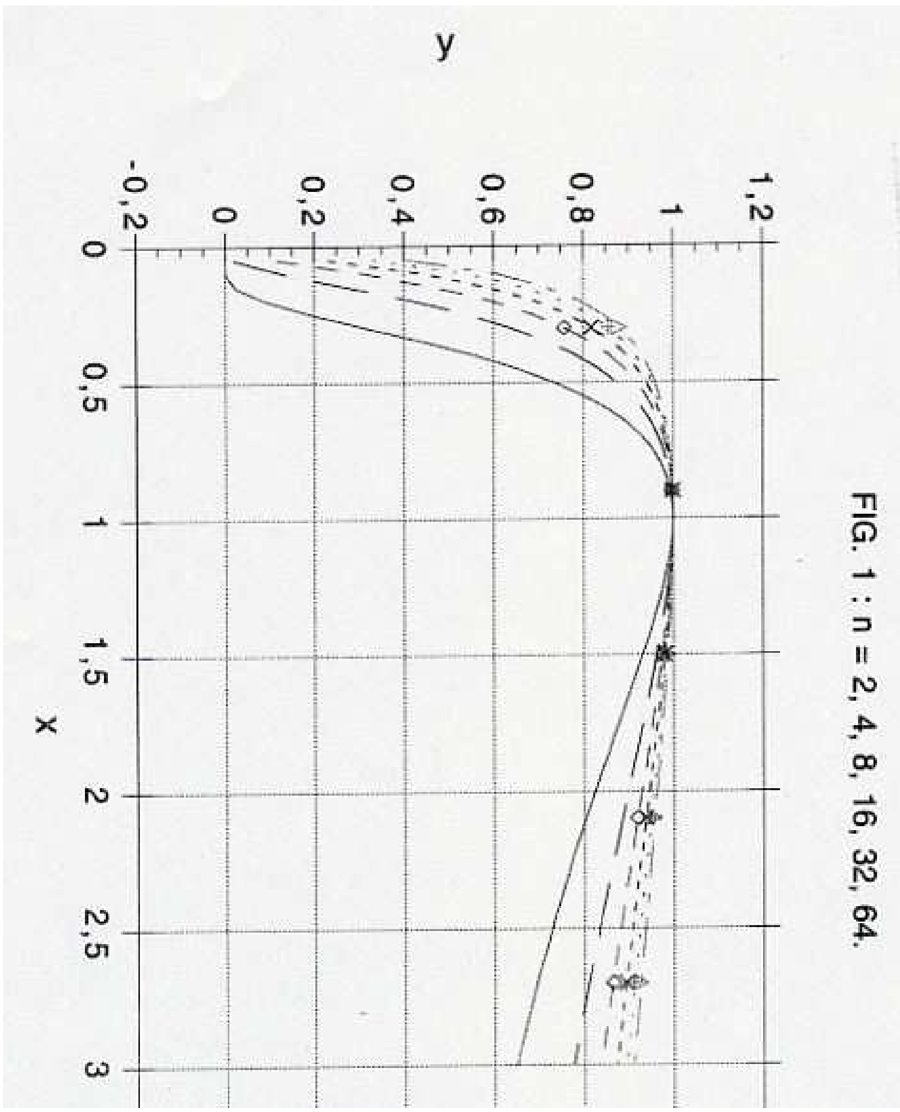

Figure 1: Generic curves $y = G_n(x)$ for $n = 2, 4, 8, 16, 32$ and 64. As one can see, the greater is n the steepest is the curve below $x = 0.5$ and the flatest it is beyond.

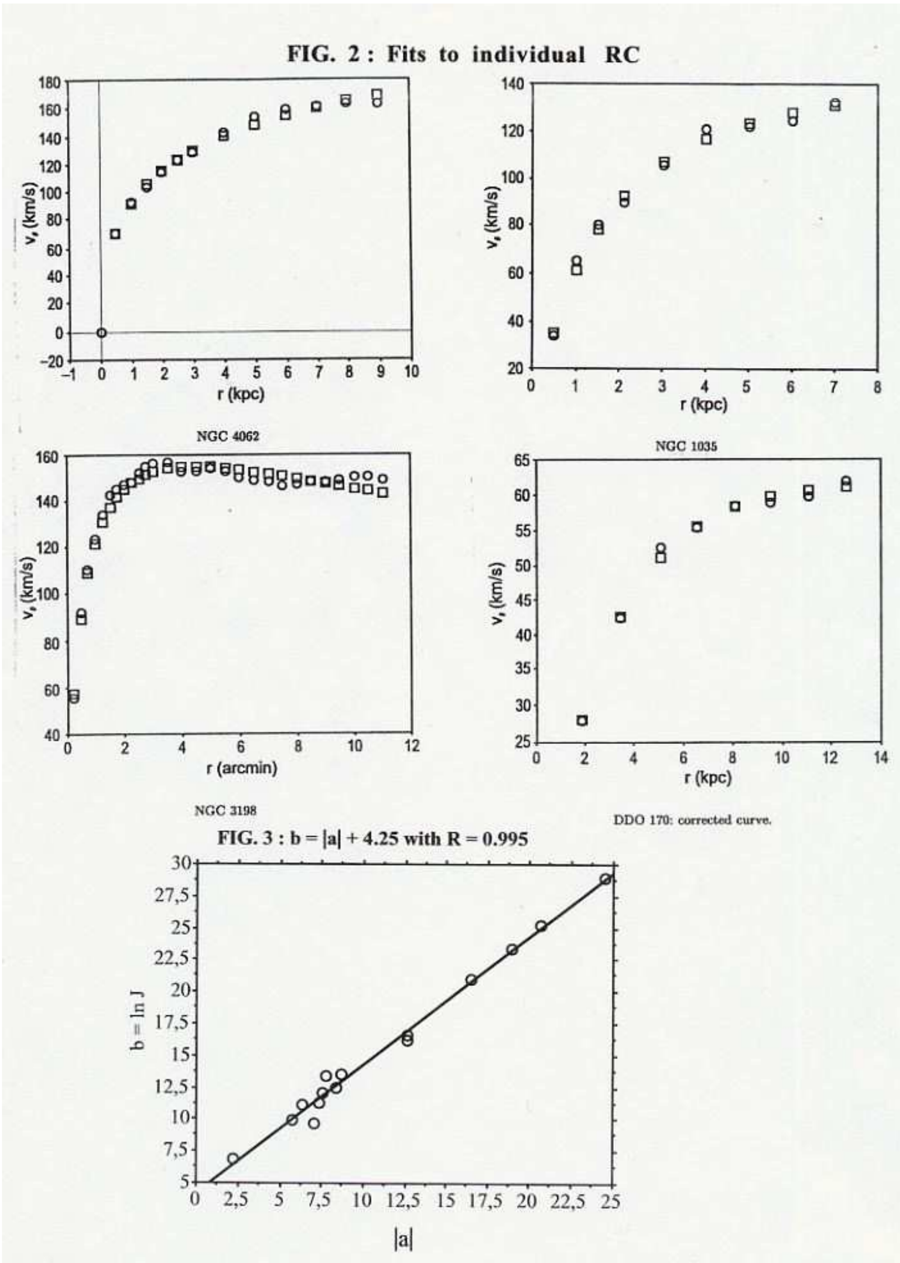

Figure 2: Rotation curve fits for NGC 4062 [2], NGC 1035 [3], NGC 3198 [4] and DDO 170 [5]. Below, in figure 3 one can see the strong linear correlation between $\mid a \mid$ and b.